PAPER - II
MATHEMATICS

41. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1,2,3,4,5 and 5, are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is

(a) $\frac{91}{216}$ (b) $\frac{108}{216}$

(c) $\frac{125}{216}$ (d) $\frac{127}{216}$

Ans. (A)

42. If P is 3×3 matrix such that P^T is the transpose of P and I is the 3×3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(a) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (b) $PX = X$

(c) $PX = 2X$ (d) $PX = -X$

Ans. (D)

43. Let $\alpha(a)$ and $\beta(a)$ be the roots the equation $\left(\sqrt[3]{1+a}-1\right)x^2 + \left(\sqrt{1+a}-1\right)x + \left(\sqrt[6]{1+a}-1\right) = 0$ where $a > -1$.

(a) $-\frac{5}{2}$ and 1 (b) $-\frac{1}{2}$ and -1

(c) $-\frac{7}{2}$ and 2 (d) $-\frac{9}{2}$ and 3

Ans. (B)

44. The equation of a Plane apssing through the line of intersection of the planes $x+2y+3z=2$ and $x-y+z=3$ and at a distance $\frac{2}{\sqrt{3}}$ form the point $(3,1,-1)$ is

(a) $5x-11y+z=17$

(b) $\sqrt{2}x+y=3\sqrt{2}-1$

(c) $x+y+z=\sqrt{3}$

(d) $x-\sqrt{2}y=1-\sqrt{2}$

Ans. (A)

45. Let $a_1, a_2, a_3....$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(a) 22 (b) 23

(c) 24 (d) 25

Ans. (D)

46. If $\vec{a}$ and $\vec{b}$ are vectors such that $\left|\vec{a}+\vec{b}\right| = \sqrt{29}$ and $\vec{a}\times\left(2\hat{i}+3\hat{j}+4\hat{k}\right) = \left(2\hat{i}+3\hat{j}+4\hat{k}\right)\times\hat{b}$, then a possible value of $\left(\vec{a}+\hat{b}\right).\left(-7\hat{i}+2\hat{j}+3\hat{k}\right)$ is

(a) 0 (b) 3

(c) 4 (d) 8

Ans. (C)

47. The value of the integral

$$\int_{-\pi/2}^{\pi/2}\left(x^2 + In\frac{\pi+x}{\pi-x}\right)\cos x dx$$

(a) 0 (b) $\frac{\pi^2}{2}-4$

(c) $\frac{\pi^2}{2}+4$ (d) $\frac{\pi^2}{2}$

Ans. (B)

48. Let PQR be a triangle of area Δ with $a=2, b=\frac{7}{2}$ and $c=\frac{5}{2}$, d where a,b and c and the lengths of the sides of the triangle opposite to be angles at P, Q, and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(a) $\frac{3}{4\Delta}$ (b) $\frac{45}{4\Delta}$ (c) $\left(\frac{3}{4\Delta}\right)^2$ (d) $\left(\frac{45}{4\Delta}\right)^2$

Ans. (C)

Paragraph for Questions 49 and 50

Let a_n denote the number of all n-digit positive fromed by the digits 0,1 or both such that no consecutive digits in thems are 0. Let $b_n =$ the number of such n-digit integers ending with digit 1 and $c_n =$ the number of such n-digit integers ending with digit 0.

49. The value of b_6 is

(a) 7 (b) 8
(c) 9 (d) 11

ANS. (B)

50. Which of the following is correct ?

(a) $a_{17} = a_{16} + a_{15}$ (b) $c_{17} \ne c_{16} + c_{15}$
(c) $b_{17} \ne b_{16} + c_{15}$ (d) $a_{17} = c_{17} + b_{16}$

ANS. (A)

Pragraph for Questions 51 to 52

A tangent PT is drawn to the circle $x^2 + y^2 = 4$ all the point $P\left(\sqrt{3},1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $\left(x-3\right)^2 + y^2 = 1$.

51. A common tangent of the two circles is

(a) $x = 4$ (b) $y = 2$
(c) $x + \sqrt{3}y = 4$ (d) $x + 2\sqrt{2}y = 6$

Ans. (D)

52. A possible equation of L is

(a) $x - \sqrt{3}y = 1$ (b) $x + \sqrt{3}y = 1$
(c) $x - \sqrt{3}y = -1$ (d) $x + \sqrt{3}y = 5$

Ans. (A)

Pragraph for Questions 53 to 54

Let $f(x) = (1-x)^2 \sin x + x^2$ for all $x \in IR$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - Int\right) f(t)dt$ for all $x \in (1,\infty)$

53. Consider the statements :

P: There exists some $x \in IR$ such that $f(x) + 2x = 2(1+x^2)$

Q: There exists some $x \in IR$ such that $2f(x) + 1 = 2x(1+x)$

Then

(a) both P and Q are true
(b) P is true and Q is false
(c) P is false and Q is true
(d) both P and Q are false

ANS. (C)

54. Which of the following true ?

(a) g is increasing on $(1,\infty)$
(b) g is decreasing on $(1,\infty)$
(c) g is increasing on (1,2) and decreasing on $(2,\infty)$
(d) g is decreasing on (1,2) and increasing on $(2,\infty)$

ANS. (B)

MORE THAN ONE MAY CORRECT TYPE

55. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0,\infty)$, then

(a) f has a local maximum at x = 2
(b) f is decreasing on (2,3)
(c) there exists some $c \in (0,\infty)$ such that $f''(c) = 0$
(d) f has a local minimum at x = 3

Ans. (A,B,C,D)

56. For every integer n, let a_n and b_n be real numbers. Let function $f : IR \to IR$ be given by

$$f(x)\begin{cases} a_n + \sin \pi x, & for \quad x \in [2n+1] \\ b_n + \cos \pi x, & for \quad x \in (2n-1, 2n) \end{cases}, \textit{for all} \text{ integers } n,$$

for all integers n. If f is continuous, then which of the following hold(s) for all n ?

(a) $a_{n-1} - b_{n-1} = 0$ (b) $a_n - b_n = 1$
(c) $a_n - b_{n+1} = 1$ (d) $a_{n-1} - b_n = -1$

Ans. (B,D)

57. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two line is (are)

(a) $y + 2z = -1$ (b) $y + z = -1$
(c) $y - z = -1$ (d) $y - 2z = -1$

Ans. (B,C)

58. If the adjoint of a 3 × 3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$,

then the possible value(s) os the determinant of P is (are)

(a) – 2 (b) – 1

(c) 1 (d) 2

ANS. (A,D)

59. Let $f:(-1,1) \to IR$ be such that $f(\cos 4\theta) = \frac{2}{2-\sec^2\theta}$ for $\theta \in \left(0,\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4},\frac{\pi}{2}\right)$. Then the value(S) of $f\left(\frac{1}{3}\right)$ is (are)

(a) $1-\sqrt{\frac{3}{2}}$ (b) $1+\sqrt{\frac{3}{2}}$

(c) $1-\sqrt{\frac{2}{3}}$ (d) $1+\sqrt{\frac{2}{3}}$

Ans. (A,B)

60. Let x and Y be two events such that $P(X/Y) = \frac{1}{2}$, $P(Y \cap Y) = \frac{1}{6}$. Which of the following is (are) correct ?

(a) $P(X \cup Y) = \frac{2}{3}$

(b) X and Y are independent

(c) X and Y are not independent

(d) $P(X^C \cap Y) = \frac{1}{3}$

Ans. (A,B)